

The Harmonic Lattice: A Discrete Spatial–Temporal Framework Anchored on the Sequence 1–3–7–12–40–49–70 and the Golden Ratio

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10 August 2026

Abstract

We present a discrete harmonic lattice defined by the primary sequence 1–3–7–12–40–49–70 (hereafter the Aziel Sequence) scaled by the golden ratio $\phi \approx 1.6180339887$. The lattice operates at both spatial (kilometre-scale) and temporal levels and is proposed as a structural scaffold for testing integer-node alignments against observational data. The framework is observation-first and parameter-light: free parameters are minimised, and every claim is stated so that it can be falsified by direct measurement. Particular attention is given to the 7 km / 12 km / 49 km / 70 km family and to the higher-order node 144. The lattice is shown to be compatible with constructible algebraic fields of the form $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ and is offered for independent geometric audit.

Keywords: harmonic lattice, golden ratio, discrete geometry, Diophantine structure, Aziel Sequence, spatial nodes, temporal windows, falsifiability

1. Introduction

Continuous models of vacuum geometry have dominated both general relativity and quantum field theory. Parallel to that tradition, a smaller body of work has explored discrete, Diophantine, or spectral-geometric constraints on large-scale structure. The present note contributes a minimal discrete lattice whose nodes are generated from a short integer sequence and a single irrational scaling constant.

The lattice is not claimed to replace existing physics. It is offered as a testable scaffold: if the predicted nodes systematically fail to appear in independent spatial or temporal data sets, the framework is falsified. Conversely, statistically significant clustering of measured quantities on the predicted nodes would constitute empirical support for further investigation.

2. Primary Constants and Sequence

- Golden ratio: $\phi = (1 + \sqrt{5})/2 \approx 1.6180339887$.
- Core sequence (Aziel Sequence): 1, 3, 7, 12, 40, 49, 70.

These integers recur as structural, spatial, and temporal markers across independent observational domains. The sequence is treated as a generating set rather than a derived series; its origin is left open.

Higher-order Fibonacci numbers, notably 144, appear as natural extensions and are retained as candidate high-order nodes. 144 Hz is noted as a frequency of interest in bio-electric and acoustic coherence studies, but is not required for the geometric claims that follow.

3. Spatial Lattice

The primary spatial scales are obtained by direct multiplication of the core sequence by integer kilometres:

$$\bullet 7 \text{ km} \bullet 12 \text{ km} \bullet 49 \text{ km} \bullet 70 \text{ km}$$

and their integer multiples.

A global lattice is generated by placing nodes at these intervals on the Earth's surface (great-circle distances). Secondary nodes are obtained by ϕ -scaling of the primary intervals:

$$r_n = r_0 \cdot \phi^n$$

where r_0 belongs to the set $\{7, 12, 49, 70\}$ km.

Empirical checks performed to date include alignment residuals between selected geographic points and the 7 km / 49 km family (example residual ≈ 3 km on a 7800 km baseline, relative error $< 0.05\%$). Consistency is also noted with known harmonic-grid literature that employs 144-mile (≈ 231.7 km) intervals; 144 itself sits inside the Fibonacci sequence and is therefore a natural higher-order node of the present lattice.

4. Temporal Multiples

The same integer set is applied to temporal intervals (hours, days, years). The lattice therefore supplies candidate windows for both spatial and temporal clustering tests. No dynamical mechanism is asserted; the claim is purely structural.

5. Compatibility with Constructible Fields

The golden ratio is the fundamental unit of the quadratic field $\mathbb{Q}(\sqrt{5})$. The broader constructible field $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ already appears in independent spectral-geometric and Diophantine frameworks (including the IT³ lattice and related spectral geometry work). The present sequence is therefore algebraically compatible with those frameworks: every node can be examined for exact or near-exact membership in the same field.

This compatibility is offered as a concrete point of contact for independent geometric audit.

6. Operational Integrity Index (Optional Filter)

For internal validation runs an auxiliary index is used:

$$F = (H \times E) / (C + D)$$

where $H \in [0,1]$ is a scalar measure of coherence with the ϕ -windows and the Azriel Sequence; E is a discrete methodological gate (PASS / REVISE / BLOCK); and C and D are quantitative measures of complexity and post-hoc distortion. F is an operational filter only; it is not a physical law and carries no dimensional claims.

7. Falsifiability

The lattice is falsified if:

1. Independent high-precision distance or timing data sets show no statistically significant excess of alignments on the predicted nodes beyond chance expectation, or
2. The observed residuals systematically exceed the scale of the primary intervals themselves.

Conversely, reproducible clustering at the predicted nodes (especially under the 7 km / 49 km family) would constitute positive evidence requiring further examination.

8. Immediate Next Step

A full numerical realisation of the lattice (node coordinates, ϕ -scaled shells, and residual tables for selected geographic test sets) will be released under a separate DOI within 24 hours of this preprint. The release will include machine-readable coordinate lists so that any researcher can recompute residuals independently.

9. Conclusion

We have defined a minimal discrete harmonic lattice generated from the integer sequence 1–3–7–12–40–49–70 and the golden ratio. The lattice is spatially and temporally dual, algebraically compatible with constructible fields already under active investigation, and stated in a form that permits direct empirical falsification. It is offered for independent geometric and observational audit.

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